OPTIMIZING THE DECISION TO TRACK IN AN AUTOMATIC RADAR PROCESSOR

HOWARD R. HENN

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Submitted in partial fulfillment of
the requirements for the degree of

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ABSTRACT

Computers and radars working together represent an emerging class of fully-automatic, high data rate radar systems. To maintain the system data rate, only a single illumination of each element of a surveillance volume is accomplished per scan in some cases. Suspected target volumes are subjected to additional illuminations for verification and acquisition into track. The basic signal detection theory is reviewed and several verification techniques are presented It is shown that, in these systems, the decision function can be optimized. For the representative examples considered, the variable threshold scheme minimizes required average power and false alarm rate in a fixed data rate system.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. Introduction.

The use of computers in radar systems can provide a fully automatic search and tracking capability. This paper is concerned with optimizing the decision to track in a rather narrow class of such systems.

One method of extending radar surveillance volume to provide adequate reaction time in defense against high velocity threats is the use of high gain antennas. The resultant narrow beamwidth, in addition to improving system accuracies, generates a requirement to rapidly step the beam through the search volume if system data rates are to be maintained. In many cases, there is not sufficient time to integrate over several pulse periods for each beam volume. There results a system whose beam is programmed to dwell for only one pulse period in the search mode in addition to updating of current tracks and the investigation of possible threats. The desire here is to determine an optimum process which provides a decision to track, or to disregard, a suspected target in a narrow beam, high data rate, single pulse per beam position (i.e., single "hit per look"), automatic system.

The term "acquisition" as used herein means the acquiring of a target for the system to track. Acquisition occurs when the decision mechanism receives the proper combination of outputs from additional observations programmed into a suspected target volume. There is a calculable probability that any given decision to track will, or will not, be correct. Optimizing the decision process includes maximizing the probability of a correct decision and a probability of acquisition may be defined.

Conversely, the chance that noise will be entered into track as a target may be defined as a probability of false acquisition. The reader is enjoined to note the following definitions for use in subsequent sections:

P_d = the probability of detection of signal plus noise for any given single radar pulse

P_N = the probability that noise alone will be detected as signal plus noise for any given single radar pulse

P_a = the probability of acquisition; the probability that a target will be correctly detected and entered into track

P_f = the probability of false acquisition; the probability that noise alone will be entered into track

The probability of detection associated with video integration in a fan beam radar would here be defined as a probability of acquisition.

By choice, optimizing of the decision process is considered as a function of required data rate, average power requirements, and equipment requirements necessary to process and store information concerning suspected targets. A time limited system is implied as soon as a range of threat velocities is defined. Any system has its power limit and some point at which its processing capability becomes saturated. In general, a number of radar system functions are activated if a target is decided upon; therefore, it is in the interests of maximizing probability of acquisition and data rate, and minimizing power and equipment requirements that an optimum decision process be determined. With this in mind, this paper considers single look, power limited systems, and those decision techniques that apply.

The paragraphs which follow contain three major areas. One, there

is a brief review of necessary terminology and signal detection mathematics to allow the reader to follow the material without reference to additional papers. Two, a number of decision or verification techniques are presented. Three, examples are given in which the various data rates, power requirements, etc., are tabulated and compared. These examples are considered representative and well within the current state of the art. It is felt that the approach used here is novel and one which will receive more emphasis in the future.

2. Discussion.

In the automatic processing of radar data, the human operator is replaced with a system which is at least as reliable in its logical abilities, if not more so, and is capable of analyzing greater densities of information at higher rates. A requirement for an observer, human or otherwise, is the ability to make decisions based upon information received. In the radar case, the observer is required to make statements concerning the presence or absence of signals at the receiver output. Spurious outputs due to noise and interference, intentional or otherwise, degrade the observer's ability to make positive statements. The statistical nature of these outputs forces one to operate within the confines of a probabilistic model. Statements made are in the form of decisions; a signal is or is not present. To these decisions is attached a probability as to their accuracy or reliability.

Signal detection, based on the Neyman-Pearson criterion for hypothesis testing, has received widespread attention in the literature. A hypothesis concerning the presence of noise alone as opposed to signal plus noise is rejected or accepted dependent upon whether or not the ratio of the distribution functions of signal plus noise to noise alone exceeds a chosen constant. Associated with each choice is a possible error, viz.false detections due to large noise pulses and non-detection of signals due to small signal plus noise pulses. The chosen constant is the threshold which controls the number of false detections. If the radar parameters have been adjusted to provide some desired "reliability

of detection," an observer must decide upon a course of action each time that the detector threshold is exceeded. In the automatic processor, this becomes a decision as to whether or not to track. Given a system which reports that a target probably has been detected, does an optimum procedure exist in arriving at a decision to track? This then becomes a two step process; detection, followed by acquisition into track. The optimization of this process is considered in following paragraphs. First, however, the basic detection philosophy and associated mathematics are briefly reviewed for clarity and further reference.

3. Detection Mathematics.

The detection process is one of distinguishing signals (radar echoes) plus additive gaussian noise from noise alone. Clutter and man-made interferences are not considered here. It has been shown [1][2] that thermal, shot, and cosmic noise sources are well represented by a gaussian distribution function of the form

$$dP = \frac{1}{\sqrt{2\pi\sigma_N^2}} e^{-y^2/2\sigma_N^2} dy \tag{1}$$

Such noise passed through a filter whose bandwidth is small compared to its center frequency will have an envelope described by the probability density function

$$dP = \frac{R}{\sigma_N^2} e^{-R^2/2\sigma_N^2} dR \tag{2}$$

which is the familiar Rayleigh distribution of Figure 1 where

R = voltage amplitude of the envelope

σ_N = r.m.s. noise voltage

σ_N^2 = mean square value of the noise voltage

The probability that the instantaneous noise voltage will exceed some value R_o is

$$P_N = \int_{R_o}^{\infty} \frac{R}{\sigma_N^2} e^{-R^2/2\sigma_N^2} dR = e^{-R_o^2/2\sigma_N^2} \tag{3}$$

If a detector at the output of narrow-band I.F. amplifiers is set to a threshold equal to R_o, P_N becomes the probability that there will be an output due to noise alone. Thus, P_N is the false alarm probability and

If a detector ... the output ... [illegible]

interesting quant... x_0 ... [illegible]

output due to noise alone. Thus P_N is the false alarm ... [illegible]

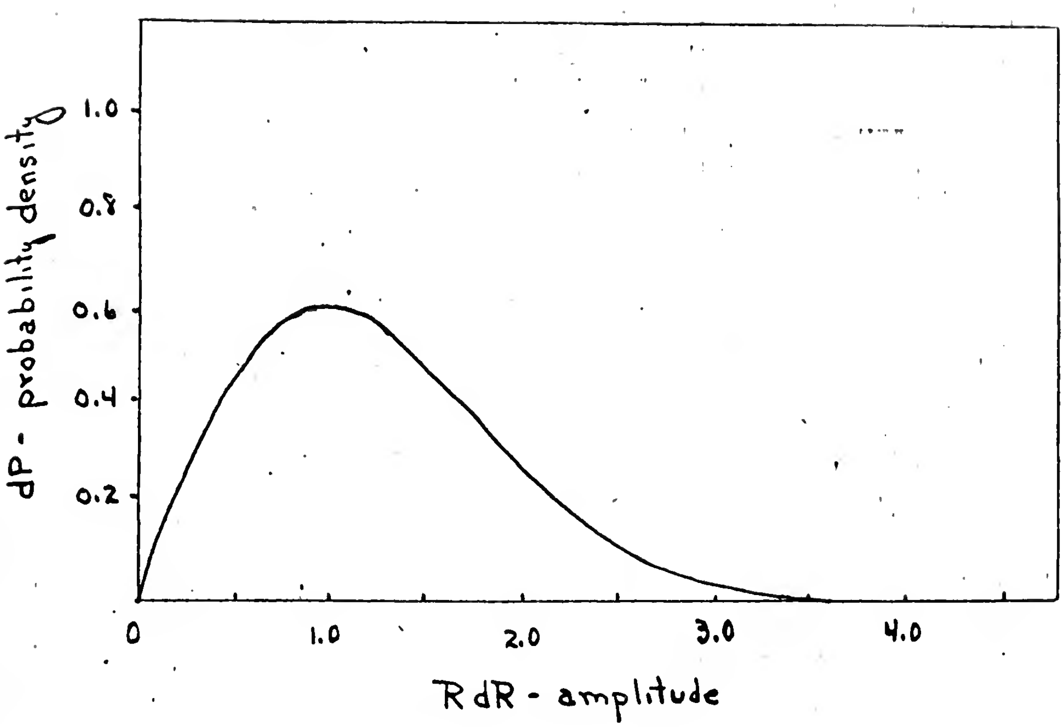


Fig. 1. Probability Density Function for the Envelope of Filtered Gaussian Noise

may be set to any predetermined value to control the fraction of time during which noise may exceed this level.

The effective input noise power to a receiver is defined to be

$$S_N = \overline{NF}kT\Delta f \tag{4}$$

where

$\overline{NF}$ = receiver noise figure

k = Boltzmann's constant, 1.38×10^{-23} joules/deg Kelvin

T = absolute receiver temperature

The minimum detectable signal is

$$S_M = S_N\left(\frac{P}{\sigma_N\sqrt{2}}\right)^2 \tag{5}$$

where $\frac{P}{\sigma_N}$ is the peak signal to r.m.s. noise voltage ratio required for adequate signal identification. For a sinusoidal signal, $\frac{P^2}{2} = \sigma_S^2$ = the mean square value of the signal voltage and the signal-to-noise ratio is $\frac{S}{N} = \frac{\sigma_S^2}{\sigma_N^2}$. Otherwise, we define the "visibility factor,"

$$f_v = \frac{P^2}{2\sigma_N^2} \tag{6}$$

The probability density function for signal plus noise at the I.F. amplifier output is

$$dP = \frac{R}{\sigma_N^2} e^{-\frac{R^2+P^2}{2\sigma_N^2}} I_0\left(\frac{RP}{\sigma_N^2}\right) dR \tag{7}$$

where $I_0\left(\frac{RP}{\sigma_N^2}\right)$ is the modified Bessel function of order zero and argument $\left(\frac{RP}{\sigma_N^2}\right)$. The probability of detection is just the probability that signal plus noise will exceed the threshold R_0 and is given by

$$P_d = \int_{R_0}^{\infty} \frac{R}{\sigma_N^2} e^{-\frac{R^2+P^2}{2\sigma_N^2}} I_0\left(\frac{RP}{\sigma_N^2}\right) dR \tag{8}$$

Equation (8) cannot be evaluated in closed form, but has been evaluated by W. R. Bennet and plotted by Rice [2]. For large values of f_V and a high threshold, [3]

$$P_d \doteq \int_{R_c}^{\infty} \frac{e^{-\frac{(R-P)^2}{2\sigma_N^2}}}{\sigma_N\sqrt{2\pi}}\, dR \tag{9}$$

which, after suitable manipulation, becomes

$$P_d \doteq \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{R_c - P}{\sigma_N\sqrt{2}}\right)\right] \tag{10}$$

where erf(x) is the error function of x. For example, if $\frac{R_o}{\sigma_N} = \frac{P}{\sigma_N} = 8$, $P_d = 0.5$, $f_V = 15$db, and $P_N = 10^{-14}$ which agrees with Rice's plot.

Change of variable and defining the new parameters [4]

$V = \frac{R}{\sigma_N}$ = ratio of envelope to r.m.s. noise voltage

$V_s = \frac{P}{\sigma_N}$ = ratio of peak signal to r.m.s. noise voltage

$V_c = \frac{R_c}{\sigma_N}$ = ratio of threshold voltage to r.m.s. noise voltage

results in equations (3) and (8) being rewritten as

$$P_N = \int_{V_c}^{\infty} V e^{-V^2/2}\, dV = e^{-V_c^2/2} \tag{11}$$

$$P_d = \int_{V_c}^{\infty} V e^{-\frac{V^2 + V_s^2}{2}} I_o(V \cdot V_s)\, dV \tag{12}$$

Substitution of equation (11) into equation (12) yields

$$P_d = \int_{\sqrt{-2\log_e P_N}}^{\infty} V e^{-\frac{V^2 + V_s^2}{2}} I_o(V \cdot V_s)\, dV \tag{13}$$

which, for a single pulse, is independent of the type of envelope detector used. This integral has been plotted by Schwartz [5] and is shown in

Figure 2. For a given value of P_d, the required signal-to-noise ratio increases extremely slowly with decreasing P_N (increasing threshold level). For fixed P_d, variation of P_N from 10^{-5} to 10^{-10} requires only a 2db increase in $\frac{S}{N}$. However, changes in P_d for a fixed P_N require large changes in power. Variation of P_d from 0.5 to 0.9 for fixed P_N requires approximately the same 2db increase in $\frac{S}{N}$. Thus, changes in P_N (1 or 2 orders of magnitude) require very little change in signal energy received.

In practice, a false alarm time interval acceptable to the system determines threshold setting. The desired probability of detection then determines the required signal-to-noise ratio to be used in the radar range equation. The probability of false alarm, P_N, and the desired false alarm time are related as [4]

$$P_N = e^{-V_c^2/2} = \frac{\text{duration of noise pulse}}{\text{interval when noise can appear}} \tag{14}$$

The time duration of a noise pulse is approximately the reciprocal of the receiver bandwidth, Δf; from the sampling theorem, there will be an independent noise pulse every $\frac{1}{\Delta f}$ seconds. The interval during which noise can appear may be factored into the product of the desired false alarm time (average interval between false target identifications) and the fraction of time that the receiver is gated on. Therefore,

$$P_N = e^{-V_c^2/2} = \frac{1}{t_f f_N \Delta f} \tag{15}$$

where

Δf = receiver bandwidth

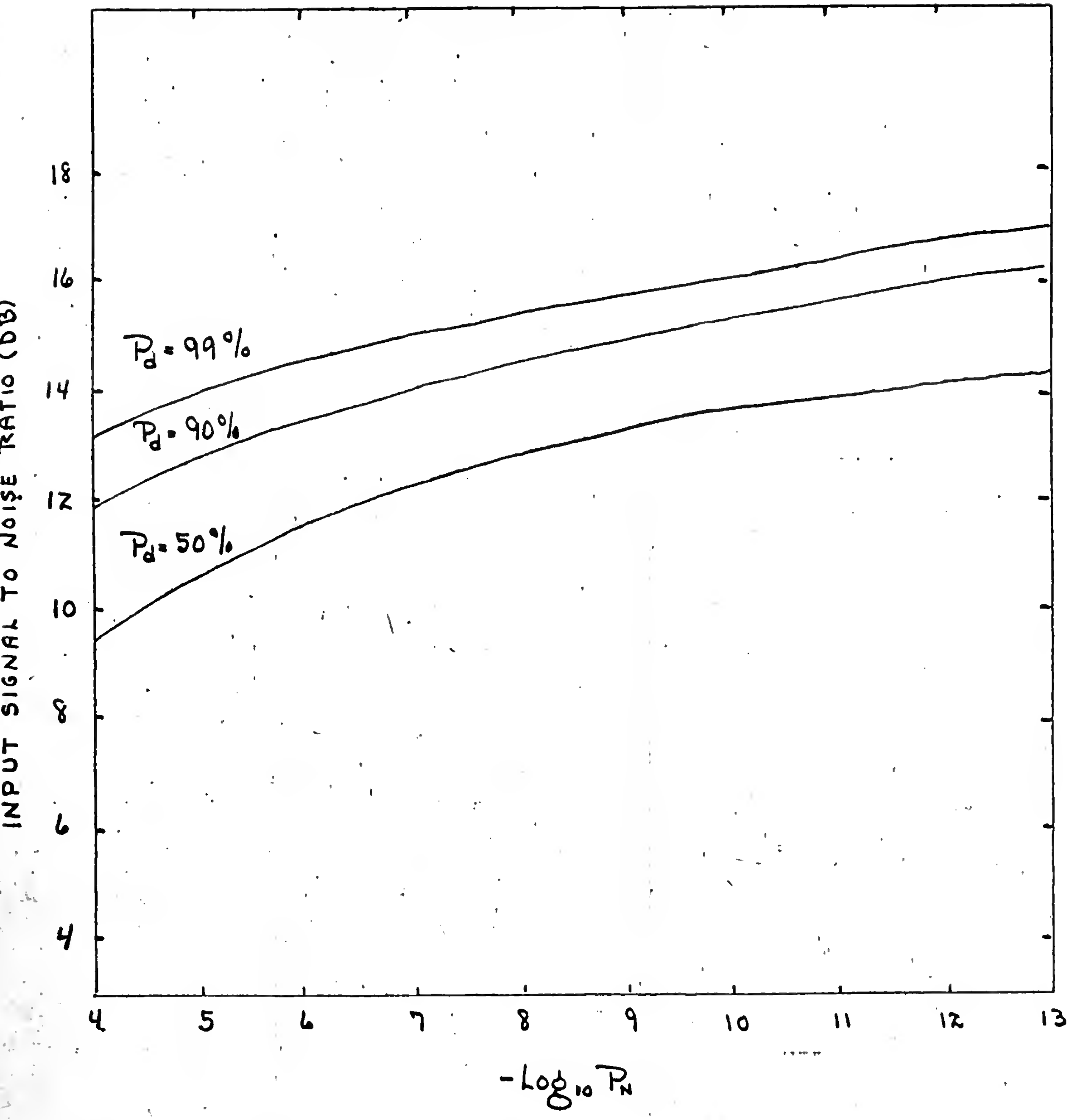


Fig. 2. Required Constant $\frac{S}{N}$ as a Function of P_d and P_N

t_f = false alarm time

f_N = fraction of time that receiver is gated on

In the absence of gating, f_N will be very nearly unity. With gating,

$$f_N = t_g f_r \tag{16}$$

where

f_r = prf

t_g = gate length in seconds

If the receiver bandwidth, Δf, is approximately $\frac{1.2}{t_d}$, where t_d is the radar pulse length, then

$$P_N = e^{-V_c^2/2} \approx \frac{t_d}{1.2\, t_f f_N} \tag{17}$$

It is now possible to calculate the detector threshold ratio, V_c, from chosen system parameters. In an automatic processor where various functions are performed at some cyclic rate, it may be advantageous to express false alarm time as

$$t_f = \frac{\text{scan time}}{\text{false alarms per scan}} \tag{18}$$

Finally, the threshold derived from equation (17) is

$$V_c = \frac{R_c}{\sigma_N} = \left[\frac{\log_{10} P_N}{-0.217}\right]^{1/2} = \left[\frac{-1}{0.217} \log_{10}\left(\frac{t_d}{1.2\, t_f f_N}\right)\right]^{1/2} \tag{19}$$

The computed value of V_c becomes the abscissa of Figure 3. The curve labeled $V_s = 0$ represents the probability density function for noise alone. The area under this curve and to the right of V_c is equal to P_N.

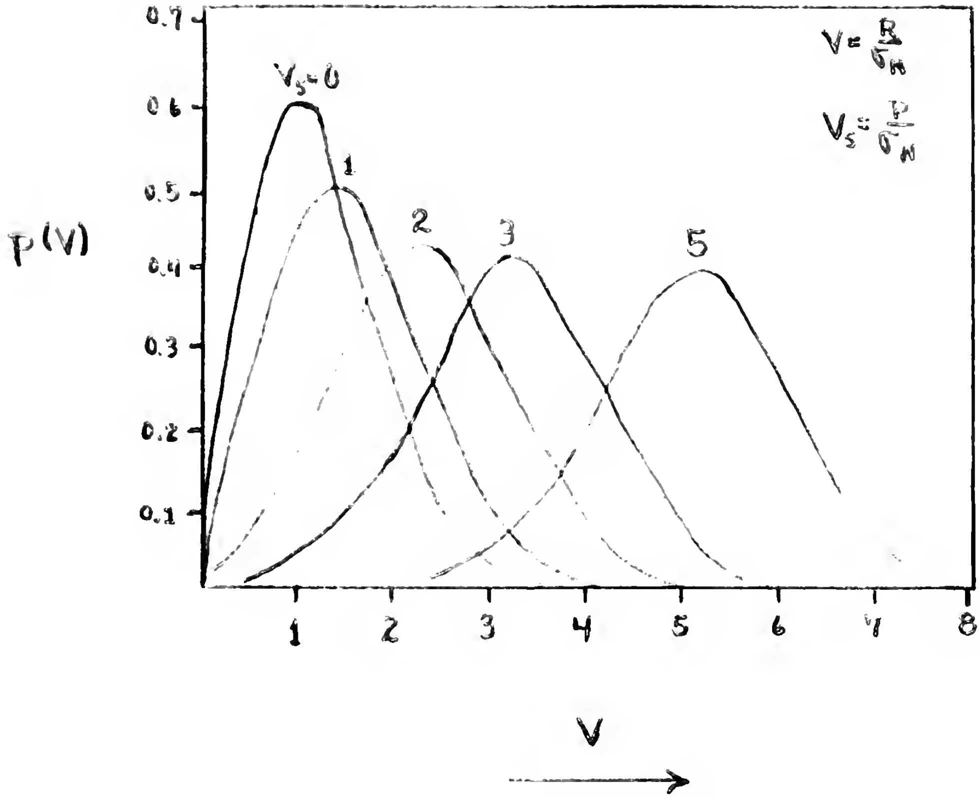


Fig. 3 - Probability Density of Envelope of Signal Plus Noise [2]

A choice of V_S, based now upon a desired probability of detection, serves to select one "signal plus noise" curve from the family of curves. Increasing values of V_S effectively slide this curve to the right and increase the area which is to the right of the threshold V_C. From Figure 3, it is well to inquire as to the existence of an optimum choice of P_N and P_d for initial detection of signal plus noise. Since V_S is tied directly to the signal-to-noise ratio of received energy and hence to the required power, a valid determination of "best" must consider this power requirement. Decreasing the threshold V_C appears to serve the same purpose; however, the resultant increased P_N means more time and equipment to process more false alarms.

It would not appear, then, that one can say unequivocally that a specific choice of P_N and P_d is more favorable in the general case V_s, in many cases, will be determined by the power available.

Thus far, only the single pulse probabilities associated with a constant amplitude signal in noise have been described. Details of the analogue integration of n pulses are adequately documented [1][5][6] and will be further referenced as required; other multiple pulse techniques will be developed. Actual signals of interest have a time-varying amplitude, the distribution of which lies between the extreme cases that have been subjected to rigorous analysis [1][6]. The development here continues with the verification of constant amplitude signals in noise.

4. Verification Techniques.

A. Discussion

The decision to track what appears to be a valid target is here defined to be a two-step ordered process. First, detection as the result of a single spatial illumination by which we mean a receiver output which has exceeded the threshold; and second, acquisition, which acknowledges the fact that n outputs ($n \geq 1$) have further (singly or in consort) met the system requirements for target declaration and entering into track. This latter function is a pre-track procedure which verifies or rejects initial detections, thus reducing the probability of erroneous reactions within the system. In addition to the probabilities of false alarm and detection, probabilities of acquisition and false acquisition may be defined as

P_a = probability that a target will be entered into track = $\Phi\left[P_d(n), n\right]$ (20)

P_f = probability that noise will be erroneously entered into track = $\Psi\left[P_n(n), n\right]$ (21)

Fan-beam radars with human observers demonstrate a pre-track procedure where multiple hits upon a target provide signal build-up on CRT displays. Several "detections" occur and the observer-display integration process results in a signal which meets the reliability requirement for acquisition. However, the first requirement in this chain of events is the reception of sufficient signal energy to exceed the detector threshold when a target is actually present. Now, the reliability of detection is influenced only by the "energy ratio" of the received signal waveform [7]. The

energy ratio is defined as the ratio of received signal energy to noise power per cycle per second of bandwidth. If the received signal energy is E_R, we can use the definition of equation (4) to determine

$$\text{energy ratio} = \frac{E_R}{\overline{NF}kT} = q\,\frac{P_t t_d}{\overline{NF}kT} = q\,\frac{P_t}{S_N} \tag{22}$$

where q is a constant determined by target range, target cross-section, and antenna gain and P_t is the peak power transmitted. From equation (22), we note that detection performance is independent of echo signal bandwidth. Accuracy, ambiguity, and resolution are determined by the actual transmitted waveform while only the energy ratio limits the detection performance available. Multiple-hit radars using the observer-display integration process require a peak power which varies approximately as $\frac{1}{\sqrt{n}}$ or, stated another way, they accrue an effective power $P_t\sqrt{n}$ where n is the number of records integrated. If non-coherent integration techniques are used, the effective peak power is $P_t b$ where $\sqrt{n} \leq b < n$ and the improvement results from the form of the n-variate distribution function for signals plus noise. For coherent integration, it can be shown that the effective peak power increases approximately as $(n - \sqrt{n})$ for large values of n $(n > 20)$.
In the light of equation (22) then, the same energy $(nP_t t_d)$ expended in a single hit system will result in a higher reliability of detection.[1]

[1] Not strictly true in all cases. For a completely noise-like signal, there are penalties attached to choices of high P_d; for example, see Fig. 3 and the related discussion in Kaplan [6].

The acquisition schemes developed herein will be examined primarily in conjunction with a narrow beam, single hit-per-look, high data rate radar. We wish to define an optimum process, or acquisition mode, for this radar.

In defining an optimum mode, we have chosen to preserve the required data rate and maintain a high probability of acquisition while concurrently minimizing average power requirements and equipment requirements incidental to the processing of false alarms. Specifically, we want to examine several verification techniques to see if there is some scheme(s) where the proper choice of P_N and P_d will yield the desired maxima and minima. The choice of P_N and P_d, in each case, will determine values of V_c and f_v (or P_t) leading to a complete set of system parameters.

Where the outputs of an automatic processor control reactions by other components of an integrated system, it is evident that false acquisition times (average time interval between erroneous entries into track) must be large. Various methods of achieving this come to mind or have been proposed [5][8][9], and amount to a confirmation of the detection criterion employed. Most of these methods use additional "looks" to confirm or reject a possible detection (threshold exceeded). The moving-window detector, which requires m detections in the n most recent "looks" at some particular range, is more appropriate in the fan-beam radar case and will not be discussed. A modified sequential observer, a success run observer [9], and fixed or variable threshold [5] schemes would have applications in a single hit-per-look system as would hybrid devices designed to reap the benefits of analogue integration; these are the cases

considered.

B. Single Look Acquisition

Undoubtedly, the simplest mechanization of an automatic processor would be that one receiving the outputs of a track-while-scan (TWS) radar. This presupposes that the basic data rate (scan rate) is adequate for tracks of interest and allows beam programming independent of any track parameters. This independence is purchased with peak power, however, since the probability of acquisition and the false acquisition time are equal to the single hit probability of detection and the false alarm time, respectively. This mode is cited since it represents a logical beginning and serves as a basis for comparison in the examples of the next section. For this mode, then,

$$P_a = P_d, \quad P_f = P_N; \quad t_f = t_{fa}$$

where t_{fa} is the false acquisition time.

C. Fixed Threshold, Multiple Look

Consider a system where confirmation is gained through repeated looks, say n-1 additional, after an initial detection. On each additional look, detection must occur at the same threshold setting. For a stationary target, which provides no enhancement due to closing range, the probability of acquisition is

$$P_a = (P_d)^n < P_d < 1 \tag{23}$$

requiring a higher basic P_d for some desired P_a. The probability of false acquisition (noise pulses entered into track) becomes

$$P_f = (P_N)^n < P_N < 1 \tag{24}$$

which is desirable. The number of false alarms (number of times that the pre-track procedure is activated) will be larger than in other schemes considered and the probability that a target does exist and will not be acquired becomes unacceptable for large n.

If the acquisition criterion is modified so that only m out of n detections ($m < n$) are required, this scheme becomes analytically the same as the moving-window detector. The probability that exactly j detections will occur is

$$P_d(j) = \frac{n!}{j!(n-j)!} P_d^{\,j}(1 - P_d)^{n-j} \tag{25}$$

and the probability of acquisition becomes

$$P_a = \sum_{j=m}^{n} \frac{n!}{j!(n-j)!} P_d^{\,j}(1 - P_d)^{n-j} \tag{26}$$

The sum in equation (26) is tabulated as the incomplete Beta function by Pearson for various values of the parameters; substitution in (25) and (26) of P_N for P_d yields the false detection and false acquisition probabilities. It has been shown that this method is 1 to 2 db down from the performance of an optimum video integrator [5][6][8] .

D. Variable Threshold, Multiple Look

The choice of a reasonable false alarm rate (rate at which pre-track procedure is activated) leads to long false acquisition times in the fixed threshold, multiple look scheme of the preceding paragraphs. Fortunately, false acquisition times on the order of days or weeks are not always mandatory, but a high probability of acquisition in an automatic system will be required. Threshold reduction for looks subsequent to the first results

in a method which maintains a relatively long false alarm time and a desired probability of acquisition. The probabilities of acquisition and false acquisition become

$$P_a = \prod_{i=1}^{n} P_d(i) \tag{27}$$

$$P_f = \prod_{i=1}^{n} P_N(i) \tag{28}$$

where the index i is associated with the i^{th} look. Here, an optimum choice of n would be sought.

To demonstrate the variable threshold concept, suppose a system had the design parameters

$$P_a = 0.75 \qquad P_f = 10^{-12}$$

and employed a "second look" process for its decision to track The peak power and detector threshold would be determined so that, with a fixed threshold,

$$P_d(1) = 0.866 = P_d(2)$$

$$P_N(1) = 10^{-6} = P_N(2)$$

It can easily be determined from any fairly accurate set of curves, [1][2][4][5][6] that for the same peak power, one combination of variable threshold settings could be

$$P_d(1) = 0.79 \qquad P_d(2) = 0.95$$

$$P_N(1) = 10^{-7} \qquad P_N(2) = 10^{-5}$$

The significant feature demonstrated is that the variable threshold system has a false alarm rate which is only one tenth that of the fixed threshold

system. If a constant data rate were required, the variable threshold system would require less average power to operate and possibly less hardware for processing false alarms.

E. Modified Sequential Observers

The idea of sequential detection has been investigated by several authors [5][10][11][12] and could lead to fewer "looks" per acquisition in a system which uses additional "looks" as a pre-track procedure. However, this is misleading as this scheme also leads to shorter false alarm times which means more "looks" to reject noise alone. Two thresholds are set; an upper level V_b and a lower level V_a. Then, define for any "look,"

$P_{sb} = P(S + N > V_b)$ = single look probability of acquisition

$P_{sa} = P(S + N < V_a)$ = single look probability that signal plus noise will not be detected

$P_{ins} = P(V_a < S + N < V_b) = 1 - (P_{sb} + P_{sa})$ = probability that signal plus noise will be detected but not acquired into track

$P_{nb} = P(N > V_b)$ = single look probability that noise will be erroneously entered into track

$P_{na} = P(N < V_a)$ = single look probability that noise will be rejected

$P_{inn} = P(V_a < N < V_b)$ = single look probability of false alarm

Any output which exceeds the upper threshold, V_b, will be considered a target and entered into track with no further pre-track procedure. An observation which results in a detection (lower threshold exceeded) but not acquisition will require another "look" to confirm or reject the hypothesis that a signal is present. The theory predicts [13] that a sufficient

number of observations will result in an output which is either greater than the upper threshold or less than the lower threshold; a decision will have been made after M observations for some M. However, the possibility of M increasing indefinitely in an automatic system cannot be tolerated. Therefore, a limit should be set on M and disposition made of signals still between the two thresholds, V_a and V_b, after M looks.

Suppose we have modified a true sequential observer so that the pre-track procedure terminates after M observations with signals still between V_b and V_a entered into track at that time. The choice of P_{nb} sets V_b, the choice of P_{sb} determines the required signal-to-noise ratio or visibility factor as in previous examples, and the choice of any of the other four probabilities sets V_a and determines the remaining three probabilities. We would like to draw some conclusions as to the average number of observations required for acquisition and the time spent in rejecting false alarms.

The probability that a signal plus noise will exceed V_b after the j^{th} look is

$$P_d(j) = (P_{ins})^{j-1} P_{sb} \tag{29}$$

while the probability that an actual signal will eventually be tracked is

$$P_a = \left[\sum_{j=1}^{M-1} (P_{ins})^{j-1} P_{sb}\right] + (P_{ins})^{M-1} (P_{ins} + P_{sb})$$

$$= P_{sb} \cdot \left[\frac{1 - P_{ins}^{M}}{1 - P_{ins}}\right] + P_{ins}^{M} \tag{30}$$

Similarly, the probability that noise alone will eventually be erroneously

tracked is

$$P_f = \left[\sum_{j=1}^{M-1} (P_{inn})^{j-1} P_{nb} \right] + (P_{inn})^{M-1} (P_{inn} + P_{nb})$$

$$= P_{nb} \cdot \left[\frac{1 - P_{inn}^{M}}{1 - P_{inn}} \right] + P_{inn}^{M} \tag{31}$$

The approximations listed previously [2] for probabilities of signal plus noise exceeding some threshold, e.g. equations (9) and (10), are only valid if $(V_s V_c) >> 1$ and $V_s >> |V_c - V_s|$. In determining the lower threshold, the second condition does not generally hold and a series evaluation of the integral is required. This may be done in different ways [1][2][5] and has been plotted for specific values of the parameters. Marcum defines a Q-function

$$Q(a,b) = \int_b^{\infty} V e^{-\frac{1}{2}(V^2 + a^2)} I_o(aV)\,dV \tag{32}$$

which he further relates to the incomplete Toronto function. The Q-function has been tabulated [14] for a wide range of values at small intervals. Schwartz [5] derives curves based upon the Edgeworth series which are adequate for many cases. Rice [2] has plotted curves for constant V_s as a function of $(V_c - V_s)$. As an example, assume

$M = 2$ = maximum number of looks

$P_{sb} = 0.5$ = probability of single look acquisition

$P_{nb} = 10^{-11}$ = probability of single look false acquisition

This determines V_b, the upper threshold, and $\frac{S}{N}$. For the value of P_{nb},

$V_b = 7.1$; a choice of $P_{sa} = 0.01$ gives

$$1 - P_{sa} = 0.99$$

as the probability that signal plus noise will exceed the lower threshold V_a. From the curves of either Rice or Schwartz, this choice of P_{sa} yields $V_a = 4.8$. The condition $V_s \gg |V_a - V_s|$ is seen not to hold since $V_s = V_b = 7.1$ for the P_{sb} and P_{nb} already chosen. To determine the probability that noise alone will exceed the lower threshold,

$$1 - P_{na} = P(N > V_a) = e^{-V_a^2/2} = e^{-11.72} = 1.26 \times 10^{-5}$$

which now provides all the probabilities necessary to complete calculations:

$$P_{sb} = 0.5$$

$$P_{sa} = 0.01$$

$$P_{ins} = 0.49$$

$$P_{nb} = 10^{-11}$$

$$P_{na} = 1 - (1.26 \times 10^{-5})$$

$$P_{inn} = (1.26 \times 10^{-5}) - 10^{-11}$$

Substitution of proper values leads to the "probability of acquisition" for signals plus noise

$$P_a = 0.5 \left[\frac{1 - (0.49)^2}{1 - 0.49}\right] + (0.49)^2 = 0.985$$

and to the "probability of false acquisition" for noise alone

$$P_f = 10^{-11} \left[\frac{1 - \left[(1.26 \times 10^{-5}) - 10^{-11}\right]}{1 - \left[(1.26 \times 10^{-5}) - 10^{-11}\right]}\right]^2 + \left[(1.26 \times 10^{-5}) - 10^{-11}\right]^2$$

$$\doteq 1.7 \times 10^{-10}$$

To illustrate a vital point here, suppose that the choice of V_b which gave $P_{sb} = 0.5$ and $P_{nb} = 10^{-11}$ had resulted in one false acquisition per hour in some system. The addition of the lower threshold, V_a, and use of the modified sequential scheme described would make target acquisition virtually a certainty. However, there would now be one false acquisition every 4.7 minutes and the system would be required to "check out" 3500 false alarms a second. Certainly a better choice of thresholds could be made through substitution of desired values of P_{ins} and P_{inn} in equations (29) and (30). At any rate, the point to note is that any savings in "looks per acquisition" may be more than offset by "looks to reject noise alone." The expected number of targets per unit time would be a factor in any considerations.

If n, the number of observations per acquisition, is a discrete variable, then the average number of observations per acquisition is

$$\bar{n} = \frac{\sum_{J=1}^{M} J P_a(J)}{\sum_{J=1}^{M} P_a(J)} \tag{33}$$

where $P_a(J)$ is the probability of acquisition on the J^{th} observation and the sum in the denominator approaches 1 as M increases. Using equation (30)

$$\bar{n} = \frac{\left[\sum_{J=1}^{M} J(P_{ins})^{J-1} \times P_{sb}\right] + MP_{ins}^{M}}{\left[\sum_{J=1}^{M} (P_{ins})^{J-1} \times P_{sb}\right] + P_{ins}^{M}} \tag{34}$$

which reduces, by taking sums of geometric series, to [5]

$$\bar{n} = \frac{\dfrac{1 - P_{ins}^{M}}{1 - P_{ins}} + \dfrac{MP_{sa}}{P_{sb}} P_{ins}^{M}}{1 + \dfrac{P_{sa}}{P_{sb}} P_{ins}^{M}} \tag{35}$$

In the example considered, $\frac{P_{sa}}{P_{sb}} = \frac{.01}{.50} = 0.02$, $M = 2$, $P_{ins}^{M} = 0.24$ so that the average number of observations per acquisition becomes $\bar{n} = 1.52$. A 0.985 probability of acquisition coupled with this average number of required observations is certainly desirable. Since a second look requires an ability to "remember" what has been seen on the first look, this utopian acquisition probability must be weighed against increased system complexity necessary to process the false acquisitions and alarms and possibly against increased average power requirements to operate in real time

F. Multiple Sensors

The outputs of several receiver channels, each connected to its own receptor, may be examined for coincidence of detection. An advantage here would be the elimination of succeeding observations. The statistics involved are those of the incomplete beta function of equation (26). The idea of separate antennas for each channel does not seem practical for the types of systems being considered while splitting or sharing of an array by more than one receiver results in 1 to 2 db of degradation. For example, suppose a system employing one receive channel were designed around a single look $P_d = 0.75$ and $P_N = 10^{-6}$. Subsequent array splitting and the addition of a second receive channel would provide 3db less in signal to

noise ratio for each channel. Complete coincidence in the two channels would now require

$$P_d(1) = P_d(2) = 0.866$$

$$P_N(1) = P_N(2) = 10^{-3}$$

Where the single channel system required 12 5db in signal to noise ratio, each of the two channels now require a signal to noise ratio of 10.5db, approximately. Since the array splitting has resulted in 3db net decrease to each channel, a degradation of approximately 10.5 - 9 5 = 1db has resulted.

Similarly, one might divide an array equally to receive on four channels and require coincidence on two or more channels (m out of n, where $m = \frac{n}{2}$ and n = 4). To maintain the same reliability as above would require

$$P_d(i) = 0.54 \qquad P_N(i) = 4 \times 10^{-4}; \qquad i = 1,2,3,4$$

or an 8.8db input signal to noise ratio for each channel. Since the array has been quartered, a 14.8db signal to noise ratio would be required overall which represents a 2.3db degradation. These statistics are identical to Harrington's [8] binary integration and Schwartz's [5] coincidence procedure. Kaplan [6] has shown this to be as much as 2db poorer than noncoherent integration while Marcum [1] indicates approximately 0.75db loss in noncoherent integration of 4 returns as compared to coherent integration. Therefore, in this trivial example, our computed 2.3db loss relative to the single look requirement agrees quite well. In a fixed data rate system, however, one would again weigh this loss against the savings

in average power since no additional observations would be required for confirmation (by the standards we have chosen here). Also, there would be fewer false alarms than in some of the other schemes as the false alarm rate would be the desired false acquisition rate.

Melton and Karr [15] have proposed a multiple receptor detection system which tests for polarity coincidence in m channels. However, the power of this technique lies in the ability to sample at n times the signal frequency. For a gaussian noise distribution given by equation (1) and a signal of known form (sinusoidal, for instance), the probability that signal plus noise will be positive or will be negative in any one channel can be computed in a straightforward manner for each of, say, n = 16 sample points. This then becomes a series of Bernoulli trials and the expected number of coincidences is the sum of the probabilities of polarity coincidence (positive or negative) associated with each sample. The expected number of coincidences for signal plus noise as compared to the expected number of coincidences for noise alone thus varies with the input signal to noise ratio. A detection criterion (threshold) may then be set to yield the acquisition and false acquisition probabilities desired.

G. A Hybrid Detector

Many radars, in mechanizing an MTI capability, employ a delay line to store the previously received information for comparison purposes. Since all of the binary integration techniques outlined so far are admittedly less efficient than their analogue counterparts, one is tempted to try and recoup this loss when more than one observation is required for acquisition.

Referring to Figure 4, suppose a pre-track procedure were to function as

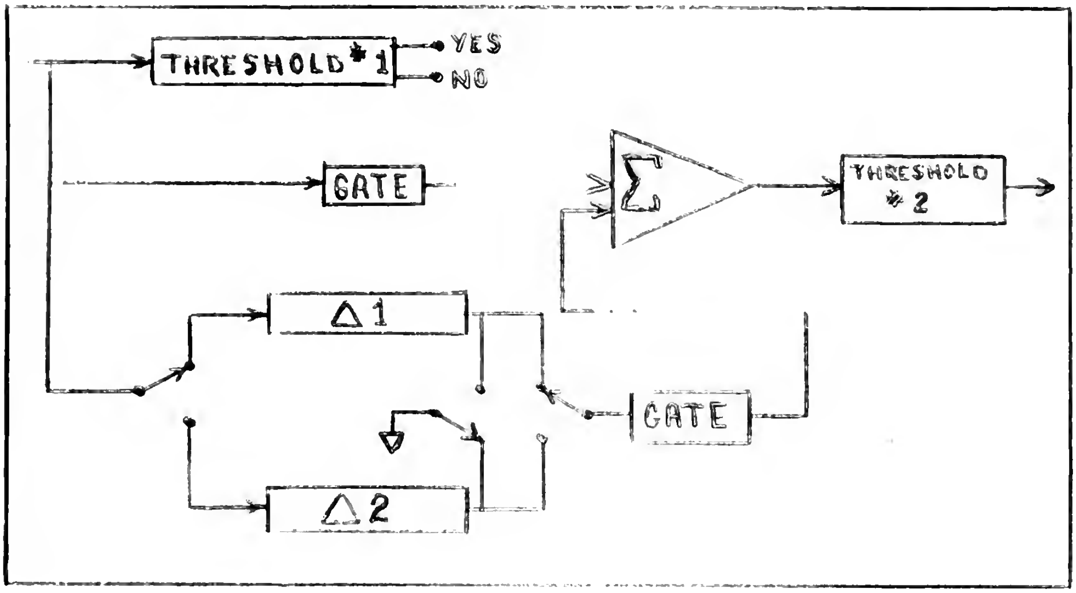


Fig. 4 - A Hybrid Detection Scheme

follows:

1. In the single look search mode, the receiver output is applied to threshold detector no. 1 and either delay circuit 1 or delay circuit 2.

2. If the receiver output exceeds threshold no. 1 (yes), switch settings do not change, the beam is programmed to the same azimuth and elevation for a second observation, and the two range gates are opened at the appropriate bin. Target declaration is now made if threshold no. 2 is exceeded.

3. If the receiver output does not exceed threshold no. 1, the switches are set to the alternate delay circuit and the beam is programmed to its next position in the scan pattern.

4. The non-used delay circuit has one interpulse period to clear

information not required.

For threshold detector no. 1, the probabilities of detection and false alarm are those developed previously in equations (11) and (12). For threshold detector no. 2, we can follow the method of Marcum [1] for the square law detector to obtain

$$P_f(n) = 1 - I\left(\frac{V_c^2}{2n}, n-1\right) \tag{36}$$

where $I(u, p)$ is the incomplete gamma function as defined and tabled by Pearson [16], and n is the number of pulses integrated. Similarly, we may use the characteristic function method to obtain the cumulative distribution for n variates of signal plus noise to obtain

$$P_a(n) = Q(V_s\sqrt{n}, V_c) + e^{-\frac{V_c^2 + nV_s^2}{2}} \cdot \sum_{r=2}^{n} \left(\frac{V_c^2}{nV_s^2}\right)^{\frac{r-1}{2}} \cdot I_{r-1}(V_c V_s \sqrt{n}) \tag{37}$$

where $Q(a,b)$ is the Q function defined in equation (32), $I_m(z)$ is the modified Bessel function of the first kind and order m, and n is again the number of pulses integrated. Now, $Q(V_s, V_c)$ is just the single look P_d; for any value of V_c, Q increases asymptotically to 1 with increasing V_s and from equation (37), the "effective" V_s increases as the square root of n. For the case considered here, then,

$$P_a = P_d(1) \times P_a(2)$$

$$P_f = P_N(1) \times P_f(2)$$

and

$$P_N(1) = e^{-\frac{V_c^2}{2}}$$

$$P_f(2) = 1 - I(0\ 3535V_c^2,\ 1)$$

$$P_d(1) = Q(V_s, V_c)$$

$$P_a(2) = Q(1.414V_s, V_c) + e^{-\frac{V_c^2 + 2V_s^2}{2}} \cdot \left(\frac{0.707V_c}{V_s}\right) I_1(1.414V_cV_s)$$

In general, the thresholds V_c associated with the two detectors may well not be the same. To reduce false alarms (false usage of pre-track procedure), the threshold associated with detector no. 1 would be comparatively high. This, in turn, raises the required signal to noise ratio for adequate signal identification and, in many cases, will result in very nearly unity P_d for the second threshold detector. For example, if $V_c = V_s = 4$ with V_c the same for both detectors, there results

$$P_N(1) = 3.36 \times 10^{-4}$$

$$P_f(2) = 3.2 \times 10^{-3}$$

$$P_d(1) = 0.5503$$

$$P_a(2) = 0.9766$$

which yields

$$P_f = 10^{-6}$$

$$P_a = 0.537$$

and the bad feature here is that there will be 336 false alarms for each false acquisition. Once again, equipment, data rate, and average power must be weighed against average number of tracks expected in any given

system. Here, however, the benefits of analogue integration are manifest; for the same false alarm rate as cited above, a variable threshold two look detector will have at most

$$P_a(2) = 0.7695$$

which reduces the acquisition probability to 0.425.

5. Examples.

Considering only the process whereby a decision is made that signal plus noise does exist at the receiver output, it is worthwhile to compare various pre-track methods for specific sets of parameters. A comparison is then available of the effect on such things as required signal to noise ratio, data rates, number of beams expended in confirmation, average power requirements, etc. Two representative systems were analyzed with parameters as follows:

All examples

receiver noise figure = 6db
two-way losses = 6db
maximum range = 200 n.mi.
dead time = 500 microseconds per sweep
tracking capacity = 400
wavelength = 0.1m.
target cross section = 1.0 sq.m.

Variables

beamwidth = 1^{o} or 2^{o} symmetric
pulselength = 5 usec at 1^{o}, 10 usec at 2^{o}
antenna gain = 44db at 1^{o}, 38db at 2^{o}
false tracks per scan = 40 at 2^{o}, 4 at 1^{o}
P_a = 0.5, 0.75, and 0.90

Additionally, an antenna consisting of four inertialess arrays was assumed with one array per quadrant of the hemispheric search volume and capabilities equally divided among the four arrays. In view of the symmetry, only one array and its search volume quadrant need be considered.

Application of the ideas contained in sections 3 and 4, in conjunction with use of the radar range equation, yields various sets of system para-

meters. In the case of the TWS system and the fixed threshold multiple-hit system, calculations are straightforward and result in a specific set of parameter values for each system. For the variable threshold multiple-hit and the modified sequential schemes, however, one can calculate several sets of parameter values which meet system requirements. Here, the use of more than one threshold provides freedom of choice for the value of at least one of the thresholds used by the system. Consequently, a true attempt at optimizing is possible. All results are contained in the tables of Figures 5 - 12.

Results for the TWS, fixed threshold, and variable threshold techniques as applied to the 2^{o} system are tabulated in Figures 5, 6, and 7 while Figures 8, 9, and 10 list the values for the same techniques as applied to the 1^{o} system. First, note that calculations have been made for several different threshold combinations in the variable threshold case. Then observe the values of the parameters we wish to optimize, viz. P_{ave}, data rate, and false alarms per scan. Here, the TWS data rate and required average power have been used as the standards to be maintained. For comparison, various possible combinations of data rate and average power requirements have been computed and listed. The number of false alarms is determined by threshold choice. Given the false alarm rate, then, the various possible combinations of P_{ave} and data rate in Figure 5, for example, are

a) best possible data rate for P_{ave} equivalent to TWS system

b) P_{ave} required for data rate equivalent to TWS system

c) best possible data rate and required P_{ave} if a multiple transmitter capability is not available; the data rate is simply the product of the required numbers of beams per scan (search, track, and process false alarms) and the pulse-to-pulse period (range time plus dead time)

Figures 6 - 10 may be interpreted in a similar manner.

Figures 11 and 12 show results of calculations for the truncated sequential observer as used in the 2^{o} system and 1^{o} system, respectively. Here again, several threshold combinations are available and the various data rates and average powers tabulated are for the same conditions listed previously. Note that the best false alarm rate is still much higher than in the variable threshold case.

Figures 13, 14,and 15 display graphically the idea that optimum threshold settings do exist for fixed data rates; i.e., optimum by the standards we have chosen. Since the minimum false alarm rate and minimum required average power do not coincide exactly, one must weigh this required P_{ave} against equipment requirements in any specific threshold choice. However, it is noted that a choice to minimize equipment requirements still leads to a P_{ave} which is very nearly equal to the best attainable minimum required P_{ave} for fixed data rate.

6. Analysis of Results.

That the shapes of the average power curves should be as outlined in Figures 13, 14, and 15 is not evident from the analytical expressions involved. The combined effects of increasing false alarm rate and fixed data rate could lead one to feel that average power was monotonically related. It is evident that such is not the case. From the tables and Figure 16, it can be seen that peak power and false alarms per scan vary inversely, as would be expected. Also, it can be seen that varying the system track capability will not effect the shapes of any of the plotted curves. Therefore, for the systems examined, there exists an optimum setting of detector thresholds; specifically, these "two-look" cases indicate the use of a variable threshold device with thresholds properly set.

If one now converts kilowatts and false alarms per scan into dollar costs, system complexity, etc., there would result a threshold pair which provided the greatest investment return. That this condition holds in the general case cannot be stated here, but the possibility merits further consideration.

7. Conclusions and Recommendations.

At this point, nothing but the broadest of generalizations can be made; admittedly, only a few examples from a small class of systems have been examined. However, the examples chosen represent realistic systems in a group currently coming into existence. The perfection of electronic scanning techniques, the advent of travelling wave tubes, and the reliability of modern, large-scale computers have all helped provide the impetus to produce these systems. The required high data rates and resulting narrow beam dimensions force one to either sector scan or abandon the traditional n target illuminations per scan. These systems do not repeatedly interrogate every volume bit; the second, third and nth looks at any point in space are programmed on a highly selective basis. Consequently, it is the initial probability of detection and probability of false alarm which become of paramount importance; it is the result obtained on one look (or some other small number) which determines the activation of a pre-track (syn., confirmation, verification) procedure. The reliabilities required in an automated system demand respectable peak powers and high thresholds; the data rates required lead to large average powers. When working with hundreds of kilowatts of average microwave power, the term "3db" must command more respect than in years passed and terms such as "1.76db" become important. For this reason, the minima of Figures 13, 14, and 15 are significant.

This work resulted from a desire to determine optimum, as compared to what often appears arbitrary, selection of thresholds. For the examples

considered, this optimum does exist in the form of a variable threshold detector. The method developed is straightforward and could be extended to other cases. Unfortunately, the investigations into all the schemes considered have consumed the time available for this work. A cursory examination indicates that a family of curves could be generated; immediately the question arises relative to the range of system parameters over which the variable threshold minima would hold. If this range is limited, will another scheme (e.g., truncated sequential or hybrid) become optimal over some other range of parameters? This answer is not immediately obvious. And is there an optimum number of additional observations to be taken in confirmation? Kaplan's work indicates one might expect such an optimum in the case of a varying signal. The m out of n criterion (binary integration), the coincidence scheme, and the double threshold receiver (where m is the second threshold) encountered in multiple-hit systems, lose their significance here and methods which exploit a priori knowledge emerge.

Average power is an explicit function of peak power, pulse width and p.r.f. By choice, p.r.f. has been a function of false alarm rate for fixed data rate and probability of acquisition. We placed no constraints on peak power. For some range, target, and receiver characteristics, one seeks to minimize the function

$$P_{ave} = G(V_s, V_{c1}, \ldots\ldots\ldots, V_{cn}, t)$$

which may be expressed explicitly, but not solved in closed form in

terms of known functions. Such an expression could be evaluated with the aid of a digital computer. The programming of the general solution of this expression could yield valuable information concerning the performance of the various acquisition schemes outlined. Dineen and Reed [9] and the Navy Electronics Lab [17] have reported efforts in this area which could be drawn upon as a starting basis.

Although definite conclusions cannot be drawn for a wide range of cases, it appears that the methods proposed are significant enough to be considered in specific applications.

	Single Look (TWS)	Single Threshold Two Looks	Variable Threshold - Two Looks			
P_N	2.62×10^{-5}	5.12×10^{-3}	a) 1.05×10^{-3} b) 2.59×10^{-2}	a) 10^{-4} b) 2.62×10^{-1}	a) 2.62×10^{-4} b) 10^{-1}	a) 3.33×10^{-3} b) 7.87×10^{-3}
P_f	2.62×10^{-5}	2.62×10^{-5}	2.7×10^{-5}	2.62×10^{-5}	2.62×10^{-5}	2.62×10^{-5}
P_d	0.75	0.866	a) 0.78 b) 0.964	a) 0.7518 b) 0.9996	a) 0.75 b) 0.996	a) 0.835 b) 0.899
P_a	0.75	0.75	0.75	0.7515	0.747	0.75
V_c	4.6	3.25	a) 3.7 b) 2.7	a) 4.3 b) 1.64	a) 4.06 b) 2.15	a) 3.39 b) 3.11
V_s	5.17	4.23	4.35	4.87	4.62	4.24
f_v	13.36 = 11.25db	8.95 = 9.52db	9.46 = 9.76db	11.86 = 10.74db	10.67 = 10.28db	8.98 = 9.53db
no. of beams	1275	2650*	1777	1413	1475	2650
P_t	9.352 MW	6.265 MW	6.622 MW	8.302 MW	7.469 MW	6.286 MW
P_{ave}	31.2 KW	a) 31.2 KW b) 43.5 KW c) 20.9 KW	a) 31.2 KW b) 30.8 KW c) 22.1 KW	a) 31.2 KW b) 30.67 KW c) 27.67 KW	a) 31.2 KW b) 28.8 KW c) 24.9 KW	a) 31.2 KW b) 43.6 KW c) 20.95 KW
Data Rate	3.825	a) 5.32 b) 3.825 c) 7.95	a) 3.785 b) 3.825 c) 5.34	a) 3.76 b) 3.825 c) 4.25	a) 3.53 b) 3.825 c) 4.44	a) 5.34 b) 3.825 c) 7.95
False Alarms	10	1955*	402	38	100	1275
False Acquisitions	10	10	10.3	10	10	10

* – With gating, a maximum of 1275 beams required to reject false alarms.

Fig. 5 TABLE I. System Parameters for 2° Symmetric Beam & Probability of Acquisition = 0.75

	Single Look (TWS)	Single Threshold Two Looks	Variable Threshold - Two Looks			
P_N	2.62×10^{-5}	5.12×10^{-3}	a) 1.05×10^{-3} b) 2.59×10^{-2}	a) 2.62×10^{-4} b) 10^{-1}	a) 10^{-4} b) 2.62×10^{-1}	a) 3.33×10^{-3} b) 7.87×10^{-3}
P_f	2.62×10^{-5}	2.62×10^{-5}	2.7×10^{-5}	2.62×10^{-5}	2.62×10^{-5}	2.62×10^{-5}
P_d	0.5	0.707	a) 0.566 b) 0.883	a) 0.514 b) 0.977	a) 0.507 b) 0.996	a) 0.660 b) 0.758
P_a	0.5	0.5	0.5	0.5	0.5	0.5
V_c	4.6	3.25	a) 3.7 b) 2.7	a) 4.06 b) 2.15	a) 4.3 b) 1.64	a) 3.39 b) 3.11
V_s	4.49	3.65	3.73	3.97	4.20	3.66
f_v	10.08 = 10.04db	6.66 = 8.24db	6.96 = 8.36db	7.88 = 8.97db	8.82 = 9.36db	6.70 = 8.26db
no. of beams	1275	2650*	1777	1475	1413	2650
P_t	7.056 MW	4.662 MW	4.782 MW	5.516 MW	6.174 MW	4.69 MW
P_{ave}	23.52 KW	a) 23.52 KW b) 32.30 KW c) 15.54 KW	a) 23.52 KW b) 22.20 KW c) 15.94 KW	a) 23.52 KW b) 21.30 KW c) 18.39 KW	a) 23.52 KW b) 22.85 KW c) 20.58 KW	a) 23.52 KW b) 32.50 KW c) 15.63 KW
Data Rate	3.825	a) 5.25 b) 3.825 c) 7.95	a) 3.62 b) 3.825 c) 5.33	a) 3.52 b) 3.825 c) 4.44	a) 3.72 b) 3.825 c) 4.25	a) 5.29 b) 3.825 c) 7.95
False Alarms	10	1955*	402	100	38	1275
False Acquisitions	10	10	10.3	10	10	10

* With gating, a maximum of 1275 beams required to reject false alarms.

Fig. 6 TABLE II. System Parameters for 2° Symmetric Beam & Probability of Acquisition = 0.50

	Single Look (TWS)	Single Threshold Two Looks	Variable Threshold - Two Looks			
P_N	2.62×10^{-5}	5.12×10^{-3}	a) 1.05×10^{-3} b) 2.59×10^{-2}	a) 2.62×10^{-4} b) 10^{-1}	a) 10^{-4} b) 2.62×10^{-1}	a) 3.33×10^{-3} b) 7.87×10^{-3}
P_f	2.62×10^{-5}	2.62×10^{-5}	2.70×10^{-5}	2.62×10^{-5}	2.62×10^{-5}	2.62×10^{-5}
P_d	0.9	0.949	a) 0.908 b) 0.991	a) 0.9027 b) 0.9994	a) 0.902 b) 0.9997	a) 0.9349 b) 0.9639
P_a	0.9	0.9	0.9	0.902	0.9	0.9
V_c	4.6	3.25	a) 3.7 b) 2.7	a) 4.06 b) 2.15	a) 4.3 b) 1.64	a) 3.39 b) 3.11
V_s	5.79	4.76	4.91	5.25	5.49	4.78
f_v	16.76 = 12.24db	11.33 = 10.54db	12.05 = 10.8db	13.78 = 11.4db	15.07 = 11.78db	11.42 = 10.58db
no. of beams	1275	2650*	1777	1475	1413	2650
P_t	11.732 MW	7.931 MW	8.435 MW	9.646 MW	10.549 MW	7.994 MW
P_{ave}	39.11 KW	a) 39.11 KW b) 55.0 KW c) 26.44 KW	a) 39.11 KW b) 39.20 KW c) 28.12 KW	a) 39.11 KW b) 37.20 KW c) 32.15 KW	a) 39.11 KW b) 39.00 KW c) 35.16 KW	a) 39.11 KW b) 55.50 KW c) 26.65 KW
Data Rate	3.825	a) 5.38 b) 3.825 c) 7.95	a) 3.83 b) 3.825 c) 5.33	a) 3.60 b) 3.825 c) 4.43	a) 3.82 b) 3.825 c) 4.25	a) 5.42 b) 3.825 c) 7.95
False Alarms	10	1955*	402	100	38	1275
False Acquisitions	10	10	10.3	10	10	10

* With gating, a maximum of 1275 beams required to reject false alarms.

Fig.7 TABLE III. System Parameters for 2° Symmetric Beam & Probability of Acquisition = 0.90

	Single Look (TWS)	Single Threshold Two Looks	Variable Threshold - Two Looks			
P_N	3.27×10^{-7}	5.72×10^{-4}	a) 10^{-6} b) 3.27×10^{-1}	a) 10^{-5} b) 3.27×10^{-2}	a) 10^{-4} b) 3.27×10^{-3}	a) 2×10^{-4} b) 1.635×10^{-3}
P_f	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}
P_d	0.75	0.866	a) 0.751 b) 0.999	a) 0.757 b) 0.997	a) 0.785 b) 0.956	a) 0.813 b) 0.924
P_a	0.75	0.75	0.75	0.754	0.75	0.75
V_c	5.43	3.86	a) 5.27 b) 1.50	a) 4.8 b) 2.62	a) 4.3 b) 3.39	a) 4.13 b) 3.59
V_s	6.05	4.86	5.86	5.40	4.98	4.91
f_v	18.30 = 12.62db	11.81 = 10.72db	17.17 = 12.34db	14.58 = 11.64 db	12.4 = 10.93db	12.05 = 10.81db
no. of beams	5100	6950	5203	5231	5506	5812
P_t	1.620 MW	1.045 MW	1.520 MW	1.290 MW	1.097 MW	1.066 MW
P_{ave}	a) 2.70 KW b) 10.80 KW	a) 2.70 KW b) 10.80 KW c) 9.52 KW	a) 2.70 KW b) 10.80 KW c) 10.35 KW d) 10.30 KW	a) 2.70 KW b) 10.80 KW c) 8.82 KW d) 8.60 KW	a) 2.70 KW b) 10.80 KW c) 7.91 KW d) 7.32 KW	a) 2.70 KW b) 10.80 KW c) 8.10 KW d) 7.12 KW
Data Rate	a) 15.30 b) 3.825	a) 13.45 b) 3.363 c) 3.825	a) 14.65 b) 3.663 c) 3.825 d) 3.905	a) 12.5 b) 3.17 c) 3.825 d) 3.925	a) 11.2 b) 2.8 c) 3.825 d) 4.13	a) 11.47 b) 2.87 c) 3.825 d) 4.36
False Alarms	1	1750	3	31	306	612
False Acquisitions	1	1	1	1	1	1

Fig. 8 TABLE IV. System Parameters for 1° Symmetric Beam & Probability of Acquisition = 0.75

	Single Look (TWS)	Single Threshold Two Looks	Variable Threshold - Two Looks			
P_N	3.27×10^{-7}	5.72×10^{-4}	a) 10^{-6} b) 3.27×10^{-1}	a) 10^{-5} b) 3.27×10^{-2}	a) 10^{-4} b) 3.27×10^{-3}	a) 2×10^{-4} b) 1.635×10^{-3}
P_f	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}
P_d	0.5	0.707	a) 0.502 b) 0.999	a) 0.510 b) 0.987	a) 0.577 b) 0.869	a) 0.617 b) 0.812
P_a	0.5	0.5	0.5	0.5	0.5	0.5
V_c	5.43	3.86	a) 5.27 b) 1.50	a) 4.80 b) 2.62	a) 4.30 b) 3.39	a) 4.13 b) 3.59
V_s	5.34	4.27	5.18	4.72	4.38	4.35
f_v	14.26 = 11.57db	9.12 = 9.60db	13.42 = 11.28db	11.14 = 10.47db	9.59 = 9.82db	9.46 = 9.76db
no. of beams	5100	6950	5203	5231	5506	5812
P_t	1.26 MW	0.806 MW	1.19 MW	0.986 MW	0.869 MW	0.837 MW
P_{ave}	a) 2.10 KW b) 8.40 KW	a) 2.10 KW b) 8.40 KW c) 7.34 KW	a) 2.10 KW b) 8.40 KW c) 8.10 KW d) 7.94 KW	a) 2.10 KW b) 8.40 KW c) 6.74 KW d) 6.58 KW	a) 2.10 KW b) 8.40 KW c) 6.25 KW d) 5.78 KW	a) 2.10 KW b) 8.40 KW c) 6.35 KW d) 5.57 KW
Data Rate	a) 15.30 b) 3.825	a) 13.30 b) 3.33 c) 3.825	a) 14.76 b) 3.69 c) 3.825 d) 3.905	a) 12.24 b) 3.06 c) 3.825 d) 3.925	a) 11.36 b) 2.84 c) 3.825 d) 4.13	a) 11.60 b) 2.90 c) 3.825 d) 4.36
False Alarms	1	1750	3	31	306	612
False Acquisitions	1	1	1	1	1	1

Fig. 9 TABLE V. System Parameters for 1° Symmetric Beam & Probability of Acquisition = 0.50

	Single Look (TWS)	Single Threshold Two Looks	Variable Threshold - Two Looks			
P_N	3.27×10^{-7}	5.72×10^{-4}	a) 10^{-6} b) 3.27×10^{-1}	a) 10^{-5} b) 3.27×10^{-2}	a) 10^{-4} b) 3.27×10^{-3}	a) 2×10^{-4} b) 1.635×10^{-3}
P_f	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}
P_d	0.9	0.949	a) 0.9 b) 1.0	a) 0.9002 b) 0.9997	a) 0.9119 b) 0.9885	a) 0.9228 b) 0.9757
P_a	0.9	0.9	0.9	0.9	0.9	0.9
V_o	5.43	3.86	a) 5.27 b) 1.50	a) 4.80 b) 2.62	a) 4.30 b) 3.39	a) 4.13 b) 3.59
V_s	6.62	5.39	6.47	5.99	5.55	5.45
f_v	21.91 = 13.40db	14.52 = 11.62db	20.93 = 13.20db	17.94 = 12.54db	15.40 = 11.87db	14.85 = 11.71db
no. of beams	5100	6950	5203	5231	5506	5812
P_t	1.94 MW	1.285 MW	1.85 MW	1.586 MW	1.36 MW	1.313 MW
P_{ave}	a) 3.24 KW b) 12.95 KW	a) 3.24 KW b) 12.95 KW c) 11.70 KW d) 8.57 KW	a) 3.24 KW b) 12.95 KW c) 12.60 KW d) 12.35 KW	a) 3.24 KW b) 12.95 KW c) 10.85 KW d) 10.57 KW	a) 3.24 KW b) 12.95 KW c) 9.78 KW d) 9.08 KW	a) 3.24 KW b) 12.95 KW c) 9.98 KW d) 8.76 KW
Data Rate	a) 15.30 b) 3.825	a) 13.80 b) 3.45 c) 3.825 d) 5.22	a) 14.85 b) 3.71 c) 3.825 d) 3.905	a) 12.80 b) 3.20 c) 3.825 d) 3.925	a) 11.55 b) 2.89 c) 3.825 d) 4.13	a) 11.80 b) 2.95 c) 3.825 d) 4.36
False Alarms	1	1750	3	31	306	612
False Acquisitions	1	1	1	1	1	1

Fig. 10 TABLE VI. System Parameters for 1° Symmetric Beam & Probability of Acquisition = 0.90

Truncated Sequential Observer ($M = 2$)

P_{nb}	10^{-6}	2.52×10^{-5}	10^{-5}
$1 - P_{na}$	5.02×10^{-3}	10^{-3}	4.02×10^{-3}
P_f	2.62×10^{-5}	2.62×10^{-5}	2.62×10^{-5}
P_{sb}	0.168	0.504	0.302
$1 - P_{sa}$	0.857	0.816	0.838
P_a	0.759	0.759	0.759
V_b	5.27	4.60	4.80
V_a	3.26	3.72	3.32
V_s	4.20	4.50	4.17
f_v	8.82 = 9.45db	10.125 = 10.06db	8.69 = 9.39db
no. of beams	2629*	1682	2610*
P_t	6.174 MW	7.088 MW	6.083 MW
P_{ave}	a) 31.20 KW b) 42.50 KW c) 20.58 KW	a) 31.20 KW b) 31.10 KW c) 23.60 KW	a) 31.20 KW b) 41.50 KW c) 20.28 KW
Data Rate	a) 5.2 b) 3.825 c) 7.91	a) 3.82 b) 3.825 c) 5.05	a) 5.09 b) 3.825 c) 7.85
False Alarms	1916	373	1532
False Acquisitions	10	10	10
$\bar{n}$	1.785	1.34	1.6

* With gating, a maximum of 1275 beams required to reject false alarms

TABLE VII. System Parameters for 2° Symmetric Beam and Probability of Acquisition = 0.759

Fig. 11

Truncated Sequential Observer ($M = 2$)

P_{nb}	10^{-7}	3.17×10^{-7}	2.27×10^{-7}	10^{-8}
$1 - P_{na}$	4.77×10^{-4}	10^{-4}	3.16×10^{-4}	5.63×10^{-4}
P_f	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}	3.27×10^{-7}
P_{sb}	0.228	0.411	0.300	0.126
$1 - P_{sa}$	0.850	0.8307	0.840	0.859
P_a	0.757	0.759	0.754	0.756
V_b	5.68	5.47	5.54	6.07
V_a	3.91	4.30	4.02	3.87
V_s	4.83	5.15	4.90	4.83
f_v	11.66 = 10.67db	13.26 = 11.22db	12.0 = 10.79db	11.66 = 10.66db
no. of beams	6630	5451	6130	6907
P_t	1.032 MW	1.174 MW	1.062 MW	1.032 MW
P_{ave}	a) 2.70 KW b) 10.80 KW c) 8.95 KW d) 6.88 KW	a) 2.70 KW b) 10.80 KW c) 8.36 KW d) 7.27 KW	a) 2.70 KW b) 10.80 KW c) 8.52 KW d) 7.07 KW	a) 2.70 KW b) 10.80 KW c) 9.31 KW d) 6.88 KW
Data Rate	a) 12.66 b) 3.17 c) 3.825 d) 4.97	a) 11.85 b) 2.96 c) 3.825 d) 4.09	a) 12.05 b) 3.01 c) 3.825 d) 4.60	a) 13.20 b) 3.30 c) 3.825 d) 5.18
False Alarms	1460	305	967	1723
False Acquisitions	1	1	1	1
$\bar{n}$	1.70	1.46	1.63	1.84

TABLE VIII. System Parameters for 1° Symmetric Beam and Probability of Acquisition = 0.75

Fig. 12

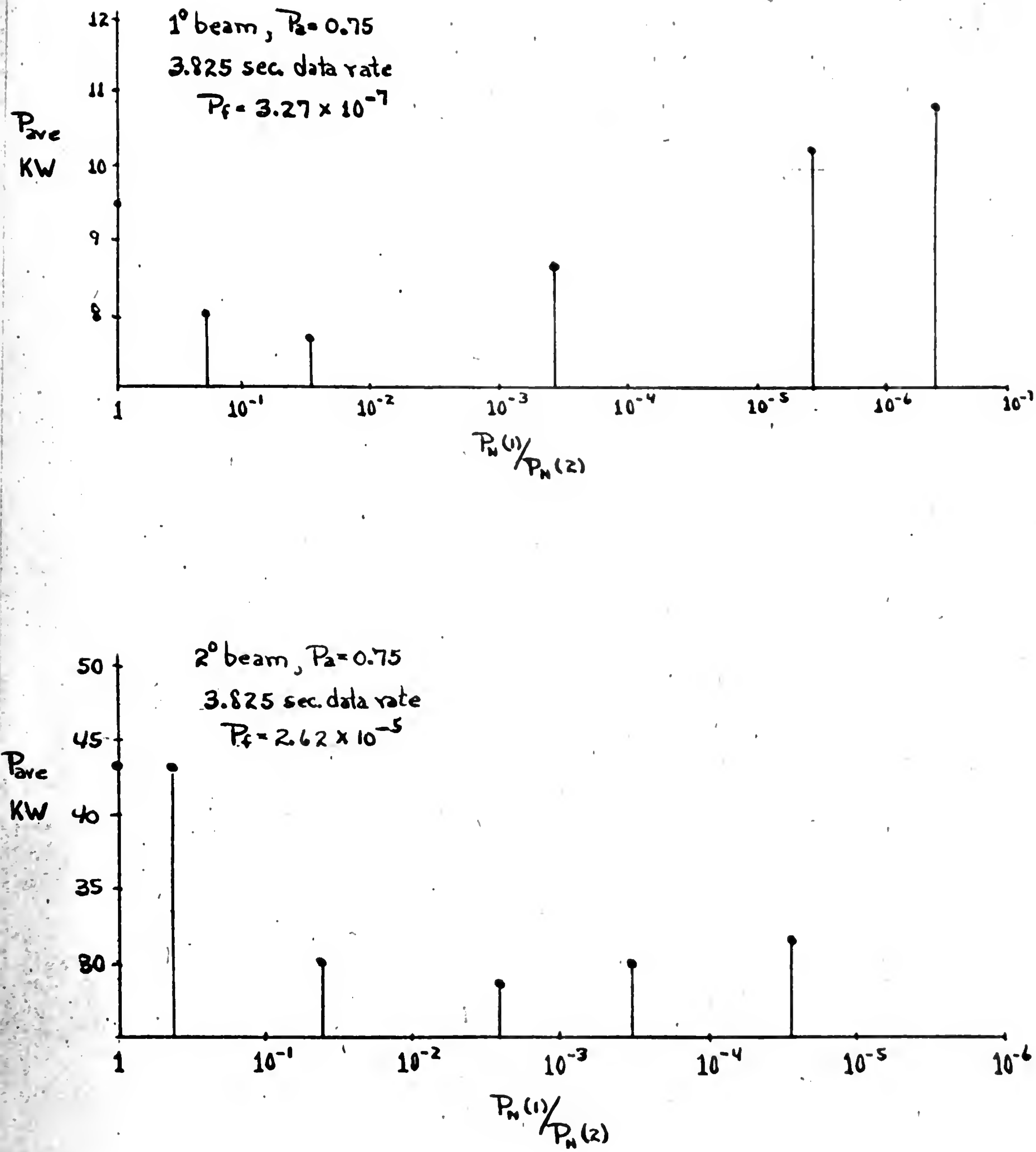
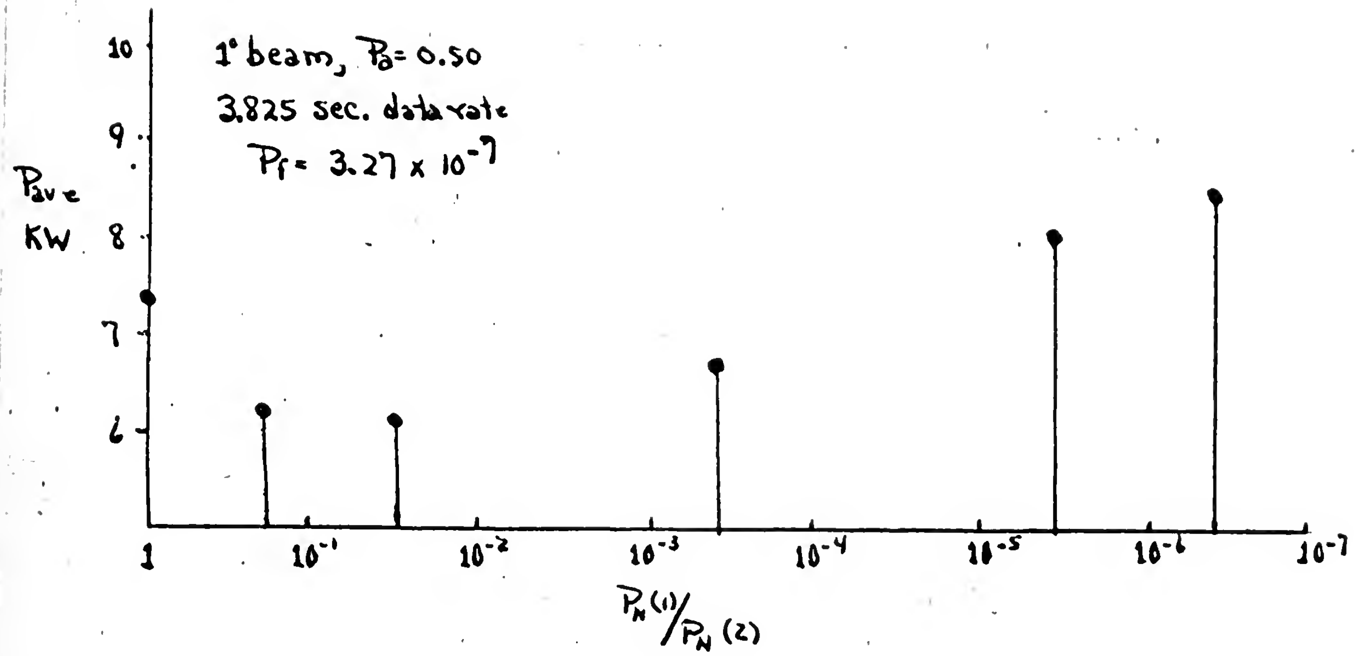


FIG 13 - AVERAGE POWER PER QUADRANT VS. THRESHOLD RATIO FOR FIXED DATA RATE



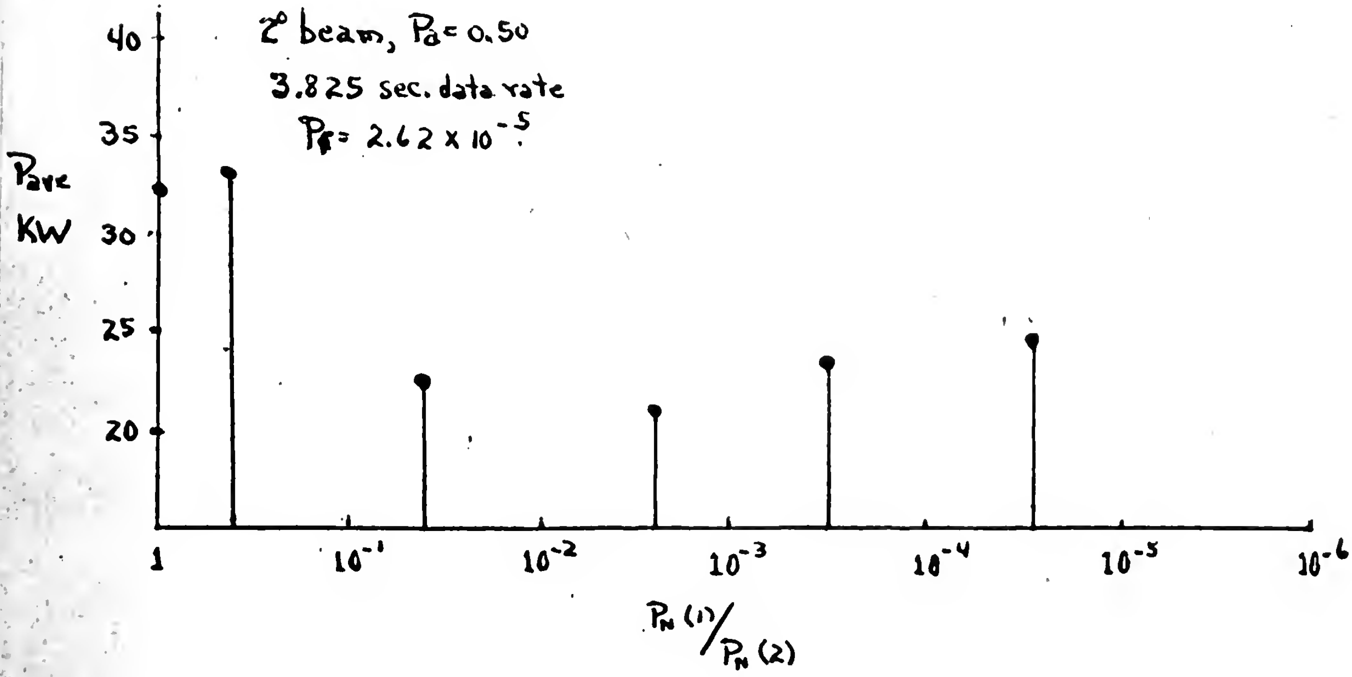


FIG 14 - AVERAGE POWER PER QUADRANT VS. THRESHOLD RATIO FOR FIXED DATA RATE

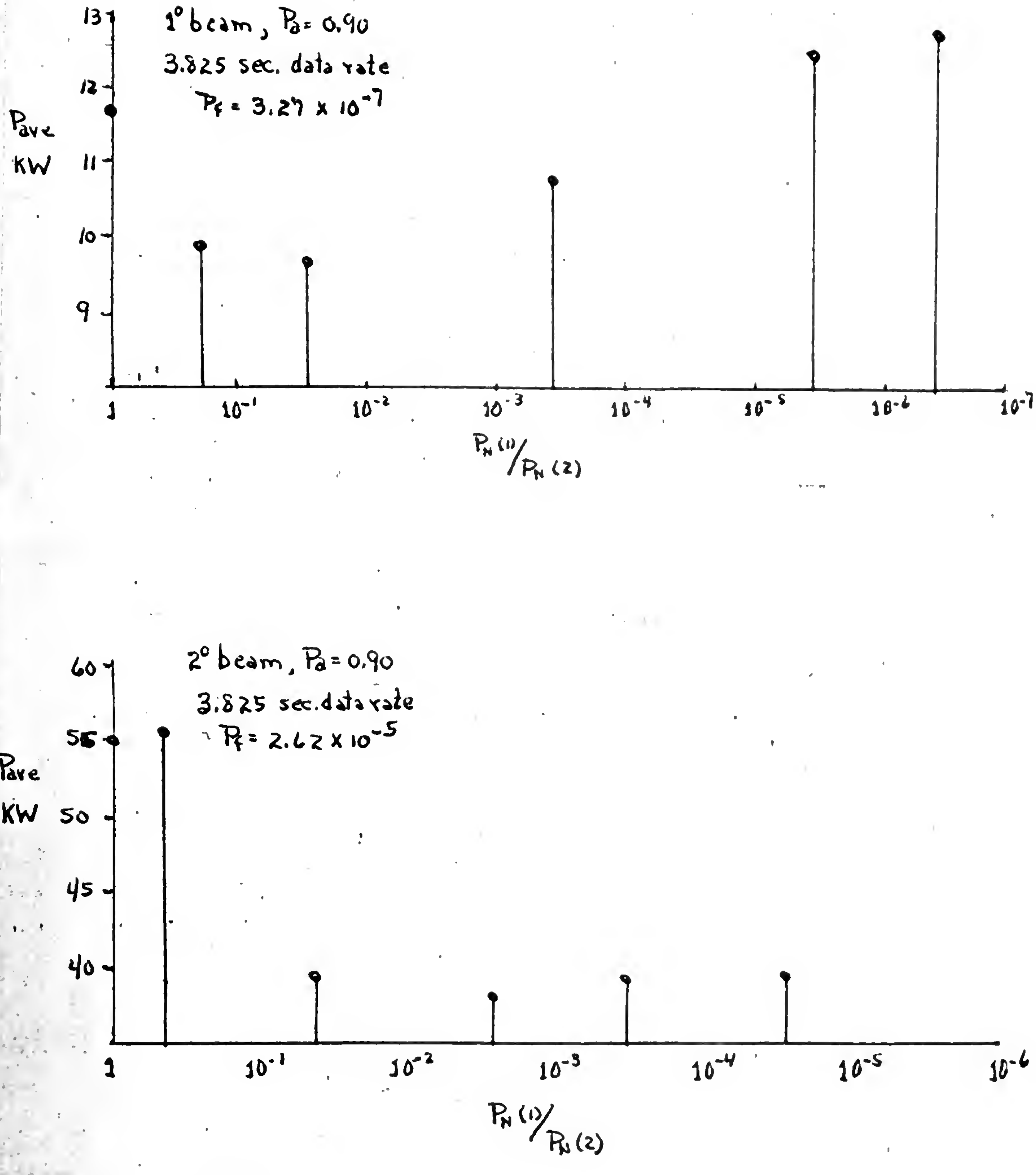


FIG. 15 - AVERAGE POWER PER QUADRANT VS. THRESHOLD RATIO FOR FIXED DATA RATE

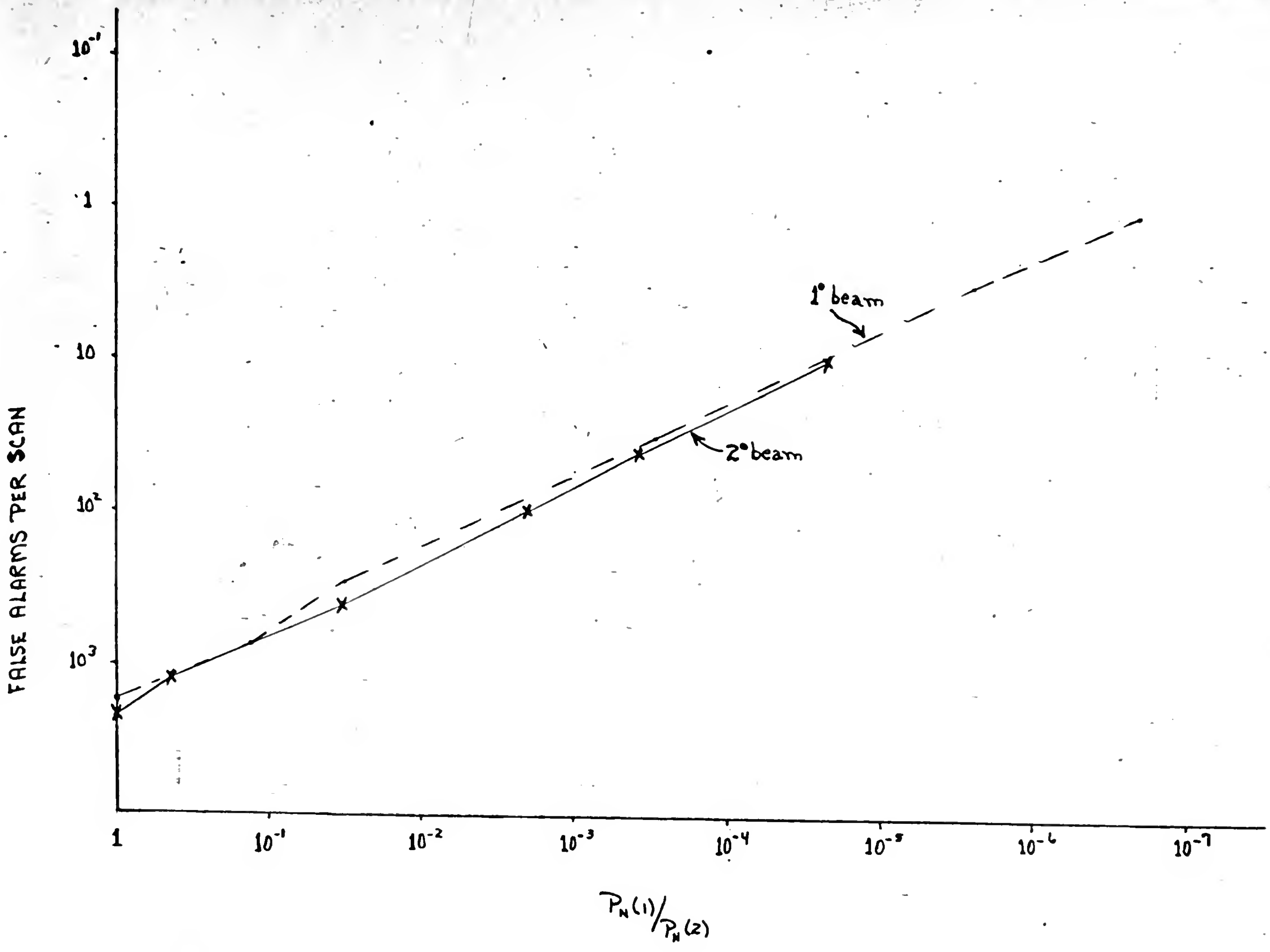


Fig. 16 - False Alarms per Quadrant Scan vs. Threshold Ratio for Fixed Data Rate

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